Eigenvalues and Eigenvectors - Problems 3

(a) Find the eigenvalues for the following matrix A, and for each eigenvalue λ of A determine a maximal set of linearly independent eigenvectors associated to λ. Say then if the matrix is diagonalizable or not, and motivate your answer. In the case A is diagonalizable, determine an invertible matrix U such that $U^{-1}AU = D$ is diagonal.

$$A = \begin{pmatrix} 8/7 & 2/7 & 1/7 \\ 4/7 & 15/7 & 4/7 \\ -2/7 & -4/7 & 5/7 \end{pmatrix}$$

(b) Find the eigenvalues for the following matrix B, and for each eigenvalue λ of B determine a maximal set of linearly independent eigenvectors associated to λ. Say then if the matrix is diagonalizable or not, and motivate your answer. In the case B is diagonalizable, determine an invertible matrix U such that $U^{-1}BU = D$ is diagonal.

$$B = \begin{pmatrix} 2 & 1 & 0 \\ -2 & -1 & 0 \\ 1/2 & 1/2 & 0 \end{pmatrix}$$

Eigenvalues and Eigenvectors - Answers 3

(a) The eigenvalues are $\lambda = 2, 1$. The eigenvalue $\lambda = 2$ has only one linearly independent eigenvector, namely $\begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$, while $\lambda = 1$ has two linearly independent eigenvectors, namely $\begin{pmatrix} -2 \\ -1 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$.

Since there are three linearly independent eigenvectors, A is diagonalizable.
A can be diagonalized by taking

$$U = \begin{pmatrix} 1 & -2 & 3 \\ 4 & -1 & -2 \\ -2 & 4 & 1 \end{pmatrix}$$

so that

$$U^{-1}AU = \begin{pmatrix} 1/7 & 2/7 & 1/7 \\ 0 & 1/7 & 2/7 \\ 2/7 & 0 & 1/7 \end{pmatrix} \begin{pmatrix} 8/7 & 2/7 & 1/7 \\ 4/7 & 15/7 & 4/7 \\ -2/7 & -4/7 & 5/7 \end{pmatrix} \begin{pmatrix} 1 & -2 & 3 \\ 4 & -1 & -2 \\ -2 & 4 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(b) The eigenvalues are $\lambda = 1, 0$. The eigenvalue $\lambda = 1$ has only one linearly independent eigenvector, namely $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$.

Also $\lambda = 0$ has only one linearly independent eigenvector, namely $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

Therefore B is not diagonalizable, since it has at most two linearly independent eigenvectors.